MWAA 2015
IU Bloomington, October 9–11th

Friday		
16:00–17:00	**F. Nazarov**	David-Semmes problem and reflectionless measures
Saturday		
09:30–10:00	**Coffee**	
10:00–10:40	**S. Denisov**	Steklov problem in the theory of orthogonal polynomials
10:50–11:30	**L. Baratchart**	Exterior asymptotics of weighted Bergman polynomials
11:40–12:20	**R. Orive**	On equilibrium problems on the real axis. Applications
12:20–14:00	**Lunch**	
14:00–14:40	**P. Gupta**	Rational density and isotropic embeddings
14:50–15:30	**Eu. Malinnikova**	Local regularity, multifractal analysis, and boundary behavior of harmonic functions
15:30–16:00	**Coffee**	
16:00–16:40	**Yu. Lyubarskii**	Uncertainty principle for discrete Schrödinger evolution
16:50–17:30	**P. Miller**	The semiclassical sine-Gordon equation and rational solutions of Painlevé-II
Sunday		
09:00–09:30	**Coffee**	
09:30–10:10	**T. Bayraktar**	Universality for zeros of random polynomials
10:20–11:00	**A. Reznikov**	Covering properties of random points
11:00–11:30	**Coffee**	
11:30–12:10	**D. Kinzebulatov**	A new approach to the L^p-theory of $-\Delta + b \cdot \nabla$, and its application to Feller processes with general drifts
12:20–13:00	**Y. Zhang**	On flat solutions to Cauchy-Riemann equations

Laurent Baratchart
INRIA, Sophia Antipolis
Exterior asymptotics of weighted Bergman polynomials

For Ω a smooth domain in the plane and $w \in L^1(\Omega)$ a non-negative weight, we consider the orthonormal polynomials P_n in $L^2(\Omega, w)$. We prove exterior asymptotics for P_n outside the convex hull of Ω under weak regularity assumptions for the weight. This generalizes previous results by Korovkin, Suetin, Miña-Díaz, and Simanek.

Turgay Bayraktar
Syracuse University
Universality for zeros of random polynomials

The *universality phenomenon* in the context of random polynomials says that asymptotic distribution of (appropriately normalized) zeros of random polynomials should become independent of the choice of distribution of random coefficients as their degree goes to infinity. In this talk, I will present some recent results on universality of limiting zero distribution of multivariate random polynomials. I will also describe generalization of these results to the setting of random holomorphic sections of high powers $L^{\bigotimes n}$ of an ample line bundle $L \to X$ over a projective manifold X.

Sergey Denisov
University of Wisconsin-Madison
The Steklov problem in the theory of orthogonal polynomials

The problem by Steklov consists in obtaining sharp bounds on the polynomials orthogonal on the unit circle (or a segment on the real line) with respect to a measure in the given class. We will discuss some recent developments which gave the complete solution to this problem. The connections to sharp estimates on the Hilbert transform in weighted spaces will be explained and some new bounds will be presented.

Purvi Gupta
University of Western Ontario
Rational density and isotropic embeddings

It is well-known that any complex-valued continuous function on the unit circle can be approximated by rational combinations of exactly one continuous function. We will discuss analogous phenomena for certain function spaces on general manifolds. This problem has interesting connections to complex analysis and symplectic geometry. Joint work with R. Shafikov.

Damir Kinzebulatov
University of Toronto

A new approach to the L^p-theory of $-\Delta + b \cdot \nabla$, and its application to Feller processes with general drifts

The problem of constructing a Feller process on $\mathbb{R}^d$, $d \geqslant 3$, having infinitesimal generator $-\Delta + b \cdot \nabla$, with a singular vector field $b : \mathbb{R}^d \to \mathbb{R}^d$ ("a diffusion with drift b") has been thoroughly studied in the literature, motivated by applications to Mathematical Physics, as well as the search for the maximal (general) class of vector fields b such that the associated process exists. This search culminated in several distinct classes of singular drifts (including $L^d + L^\infty$, the best possible result in terms of L^p spaces). I construct a process with b in a wide class of vector fields (measures), containing the classes previously known, and combining, for the first time, critical point singularities and critical hypersurface singularities.

I introduce a new method "of constructing the resolvent": the starting object is an operator-valued function, a 'candidate' for the resolvent of an operator realization $\Lambda_p(b)$ of $-\Delta + b \cdot \nabla$ generating a holomorphic C_0-semigroup in L^p (the key observation here is a link between $-\Delta + b \cdot \nabla$ and $(\lambda - \Delta)^{-\frac{1}{2}} + |b|$, as opposed to the classical approach, which compares $-\Delta + b \cdot \nabla$ to $-\Delta + b^2$). The very form of this function provides detailed information about smoothness of the *domain* $D(\Lambda_p(b))$. Now, this information about $D(\Lambda_p(b))$, combined with the Sobolev embedding theorem, allows us to move the burden of the proof of convergence in the space C_∞ of continuous functions on $\mathbb{R}^d$ vanishing at ∞ (as needed to construct the transition probability function of the process) to L^p, a space having much weaker topology (locally), hence the gain in the admissible singularities of the drift.

Yurii Lyubarskii
Norwegian University of Science and Technology

Uncertainty principle for discrete Schrödinger evolution

We prove that if a solution of a discrete time-dependent Schrödinger equation with bounded time-independent real potential decays fast at two distinct times then the solution is trivial. The continuous case was studied by L. Escauriaza, C. E. Kenig, G. Ponce, and L. Vega. We consider a semi-discrete equation, where time is continuous and spatial variables are discretized. For the free Shrödinger operator, or operators with compactly supported potential, a sharp analogue of the Hardy uncertainty principle is obtained. The argument is based on the theory of entire functions. The logarithmic convexity of weighted norms is employed for the case of general real-valued bounded potential, following the ideas developed for the continuous case. Our result for the case of a bounded potential is not optimal. Joint work with Ph. Jaming, Yu. Malinnikova, and K.-M. Perfekt.

Eugenia Malinnikova
Norwegian University of Science and Technology

Local regularity, multifractal analysis, and boundary behavior of harmonic functions

We will define local regularity of functions and survey recent results connecting the multifractal analysis to local regularity. These ideas are applied to the study of boundary behavior of harmonic functions in the upper half-space. We use wavelet characterization and martingale technique to prove the law of the iterated logarithm for oscillation of harmonic functions of controlled growth along vertical lines.

Peter Miller
University of Michigan

The semiclassical sine-Gordon equation and rational solutions of Painlevé-II

We formulate and study a class of initial-value problems for the sine-Gordon equation in the semiclassical limit. The initial data parametrizes a curve in the phase portrait of the simple pendulum, and near points where the curve crosses the separatrix, a double-scaling limit reveals a universal wave pattern constructed of superluminal kinks located in the spacetime along the real graphs of all of the rational solutions of the inhomogeneous Painlevé-II equation. The kinks collide at the real poles, and there the solution is locally described in terms of certain double-kink exact solutions of sine-Gordon.

This study naturally leads to the question of the large-degree asymptotics of the rational solutions of Painlevé-II themselves. In the time remaining we will describe recent results in this direction, including a formula for the boundary of the pole-free region, strong asymptotics valid also near poles, a weak limit formula, planar and linear densities of complex and real poles, and specialized asymptotic formulas valid near the boundary of the pole-free region. Joint work with Robert Buckingham (Cincinnati).

Fedor Nazarov
Kent State University

David-Semmes problem and reflectionless measures

We shall discuss one possible approach to the David-Semmes problem, asking to describe Borel measures μ in $\mathbb{R}^n$ such that the associated s-dimensional Riesz transform acts in $L^2(\mu)$. This approach leads naturally to several easy to state questions from classical potential theory, the answers to which are still waiting to be found. Joint work with Vladimir Eiderman, Ben Jaye, Alexander Volberg, Xavier Tolsa, and Maria Reguera.

Ramón Orive
Universidad de La Laguna

On equilibrium problems on the real axis. Applications

In this talk, we survey equilibrium problems in the presence of external fields, focusing our attention on the case where the conductor is the real axis. In addition, some recent advances and new results are presented, especially concerning the number of cuts (intervals) comprising the support of the equilibrium measure. Some important applications, in particular those related to asymptotics of Orthogonal Polynomials and Random Matrix models, are reviewed.

Alexander Reznikov
Vanderbilt University

Covering properties of random points

How many gas pumps should one build to ensure that every driver has a pump at a fixed small distance? Where should one place these pumps to minimize the number? It turns out that randomly placed points cover a regular set very well (even though they may be separated very badly). We will discuss this and related results.

Yuan Zhang
Indiana University – Purdue University Fort Wayne

On flat solutions to Cauchy-Riemann equations

Motivated by a UCP problem, we investigate the local existence of flat solutions to Cauchy-Riemann equations. We construct a germ of a smooth $\bar{\partial}$-closed $(0,1)$ form f, flat at $0 \in \mathbb{C}^n$, such that there is no flat smooth solution to $\bar{\partial}u = f$ locally. The construction also provides a family of Cauchy-Riemann equations whose minimal solutions restricted onto any proper subsets of the domain are no longer minimal with respect to any bounded plurisubharmonic weights. Joint work with Y. Liu, Z. Chen, and Y. Pan.