

MWAA 2016

IPFW, October 7-9th

Friday

15:00–16:00	Registration table	
16:00–17:00	E. Lundberg	The zeros of random harmonic polynomials

Saturday

09:00–09:30	Coffee	
09:30–09:40	Welcome	
09:40–10:20	A. Legg	Quadrature domains in complex variables
10:30–11:10	Y. Zhang	CR submanifolds with vanishing CR second fundamental forms
11:20–12:00	S. Dyachenko	Instability of steep ocean waves and whitecapping
12:00–14:00	Lunch	
14:00–14:40	Y. Yuan	On holomorphic isometries between bounded symmetric domains
14:50–15:30	B. Liu	Geometric analysis on the Diederich-Fornæss index
15:30–16:00	Break	
16:00–16:40	A. Rashkovskii	Local geodesics for plurisubharmonic functions
16:50–17:30	D. Zakharov	The dressing method and solutions to integrable systems

Sunday

09:00–09:20	Coffee	
09:20–10:00	R. Aceska	Time-evolving signal analysis
10:10–10:50	P. Boyvalenkov	Asymptotic bounds for energy of spherical designs
10:50–11:20	Break	Student Poster Display
11:20–12:00	K. Lindsay	Fekete polynomials and Julia sets

Roza Aceska

Ball State University

Time-evolving signal analysis

We discuss a new approach for processing time-evolving signals by employing a time-space trade off. The main motivation of the new dynamical sampling schemes is the possibility of compensation for insufficient spacial sampling density by involving samples obtained at later time instances. Successful reconstruction of spatially undersampled signals is directly related to establishing a frame from iterates of operators in the respective evolutionary system E . We consider the problem of finding conditions on the operators and the sampling locations so that the set of iterates forms a (scalable) frame for E .

Peter Boyvalenkov

Bulgarian Academy of Sciences

Asymptotic bounds for energy of spherical designs

Joint work with P. Dragnev (IPFW), D. Hardin (Vanderbilt University), E. Saff (Vanderbilt University), and M. Stoyanova (Sofia University, Bulgaria)

A spherical τ -design is a finite set $C \subset \mathbb{S}^{n-1}$ such that the equality

$$\frac{1}{\mu(\mathbb{S}^{n-1})} \int_{\mathbb{S}^{n-1}} f(x) d\mu(x) = \frac{1}{|C|} \sum_{x \in C} f(x)$$

holds for all polynomials $f(x) = f(x_1, x_2, \dots, x_n)$ of degree at most τ (i.e. the average of f over the set is equal to the average of f over \mathbb{S}^{n-1}).

Given an (extended real-valued) function $h(t) : [-1, 1] \rightarrow [0, +\infty]$, the h -energy of a spherical code $C \subset \mathbb{S}^{n-1}$ is given by

$$E(n, C; h) := \sum_{x, y \in C, x \neq y} h(\langle x, y \rangle),$$

where $\langle x, y \rangle$ denotes the inner product of x and y . The problems for minimizing energies are important and have many applications.

A natural problem is to estimate the potential energy of spherical designs of fixed dimension, strength and cardinality. Denote by

$$\mathcal{L}(n, N, \tau; h) := \inf\{E(n, C; h) : |C| = N, C \subset \mathbb{S}^{n-1} \text{ is a } \tau\text{-design}\}$$

and

$$\mathcal{U}(n, N, \tau; h) := \sup\{E(n, C; h) : |C| = N, C \subset \mathbb{S}^{n-1} \text{ is a } \tau\text{-design}\}$$

the minimum and the maximum possible h -energy of a spherical τ -design of N points on \mathbb{S}^{n-1} , respectively. Recently, we derived universal lower bounds on $\mathcal{L}(n, N, \tau; h)$ and upper bounds on $\mathcal{U}(n, N, \tau; h)$, which then define a strip where the energies belong.

We consider the behavior of our bounds in the asymptotic process where the strength τ is fixed, and the dimension n and the cardinality N tend simultaneously to infinity in the relation

$$(1) \quad \lim_{n, N \rightarrow \infty} \frac{N}{n^{k-1}} = \begin{cases} \frac{2}{(k-1)!} + \gamma, & \tau = 2k - 1, \\ \frac{1}{k!} + \gamma, & \tau = 2k, \end{cases}$$

(here $\gamma \geq 0$ is a constant and the terms $\frac{2}{(k-1)!}$ and $\frac{1}{k!}$ come from the Delsarte-Goethals-Seidel bound).

Theorem 1. *Let h be absolutely monotone on $[-1, 1]$ and τ be fixed. If n and N tend to infinity as in (1), then*

$$\mathcal{L}(n, N, \tau; h) \geq h(0)N^2 + o(N^2).$$

Other asymptotic bounds on $\mathcal{L}(n, N, \tau; h)$ and $\mathcal{U}(n, N, \tau; h)$ will be discussed as well and analogous results in binary Hamming spaces will be shown.

Sergey Dyachenko

University of Illinois Urbana-Champaign

Instability of steep ocean waves and whitecapping

Wave breaking in deep oceans is a challenge that still defies complete scientific understanding. Sailors know that at wind speeds of approximately 5m/sec, the random looking wind-blown surface begins to develop patches of white foam ('whitecaps') near sharply angled wave crests. We idealize such a sea locally by a family of close to maximum amplitude Stokes waves and show, using highly accurate simulation algorithms based on a conformal map representation, that perturbed Stokes waves develop the universal feature of an overturning plunging jet. We analyze both the cases when surface tension is absent and present. In the latter case, we show the plunging jet is regularized by capillary waves which rapidly become nonlinear Crapper waves in whose trough pockets whitecaps may be spawned.

Alan Legg

Indiana University Purdue University Fort Wayne

Quadrature domains in complex variables

A quadrature domain for square-integrable holomorphic functions is a domain on which integration of a function in the Bergman space coincides with a finite linear combination of point evaluations of the function and some of its derivatives. Planar quadrature domains have an elegant theory, but quadrature domains in several dimensions are not as well understood. I'll use the Bergman kernel to gain some beginning insights into how multi-dimensional quadrature domains might look.

Kathryn Lindsey

University of Chicago

Fekete polynomials and Julia sets

Joint work with M. Younsi.

We show that a nonempty, proper subset S of the complex plane is approximable (in a strong sense) by filled Julia sets of polynomials of degree at least 2 if and only if S is bounded and $\mathbb{C} \setminus \text{interior}(S)$ is connected. The proof is constructive and uses Fekete polynomials. We construct a sequence of polynomials $(P_n)_{n \in \mathbb{N}}$, where P_n has degree n , and prove that as $n \rightarrow \infty$, the approximation of the set by filled Julia sets gets arbitrarily accurate. We also prove an estimate for the rate of approximation in terms of geometric and potential theoretic quantities. I will discuss these results, as well as tell you how to find a polynomial whose filled Julia set spells out your name.

Bingquan Liu

University of California, Riverside

Geometric analysis on the Diederich-Fornæss index

Joint work with Krantz and Peloso.

Geometric analysis in differential geometry was initiated by Shing-Tung Yau and is a powerful tool in Riemannian geometry. It has been used to solve many problems in Riemannian geometry. In the field of several complex variables, it was not the most popular weapon to attack questions. One of the reasons is that many problems in the several complex variables relates to some types of differential equations of complex-valued functions which is currently not well understood. In this talk, we consider problems in the Diederich-Fornæss index with a viewpoint of geometric analysis and see what we obtain.

Erik Lundberg

Florida Atlantic University

The zeros of random harmonic polynomials

Joint work with Antonio Lerario.

For analytic polynomials p and q with $m = \deg(q) < \deg(p) = n$, how many zeros can $p(z) + q(z)$ have? A lower bound of n follows from the generalized argument principle, and A.S. Wilmschurst used Bezout's theorem to give an upper bound of n^2 . Wilmschurst conjectured that the true maximum is linear in n for m fixed. We will discuss the probabilistic version of this problem initiated by Wenbo Li and Ang Wei:

Q. How many zeros does a random harmonic polynomial have?

The answer depends on the definition of "random" and we will discuss outcomes for three different ensembles.

Alexander Rashkovskii

University of Stavanger

Local geodesics for plurisubharmonic functions

We apply technique of geodesics in the space of metrics on Kähler manifold to functions from Cegrell's classes on bounded domains of \mathbb{C}^n . Roughly speaking, a geodesic is the 'best' transformation of a given function to another one. We show that the standard energy functional is linear along geodesics, and derive some consequences (e.g., a capacity variant of Brunn-Minkowski inequality) from that.

Yuan Yuan

Syracuse University

On holomorphic isometries between bounded symmetric domains

The study of (local) holomorphic isometries between bounded symmetric domains has two motivations. One is rigidity/classification in the differential geometric point of view, in particular, the close relation to proper holomorphic maps, and the second one is from number theory. I will describe the recent developments on the extension and the rigidity for such maps and explain the connection to complex analysis, CR geometry and etc. The main results are joint works with M. Xiao on holomorphic isometries from the complex unit ball to type IV classical domains.

Dmitry Zakharov

Courant Institute of Mathematical Sciences

The dressing method and solutions to integrable systems

The dressing method is a way of constructing exact solutions to nonlinear integrable systems starting from their Lax representation. The input data is a \bar{d} -bar problem that specifies the analytic behavior of the joint eigenfunction of the Lax operators. This method can be viewed as a nonlinear version of the D'Alembert method of solving the wave equation, and produces solutions locally in space and time. I will explain how the dressing method can be used to easily construct the soliton solutions of the KdV equation, and can also be used to study the closure of the set of soliton solutions. I will also describe a construction of a set of soliton-like solutions of the KP equation.

Yuan Zhang

Indiana University-Purdue University Fort Wayne

CR submanifolds with vanishing CR second fundamental forms

Joint work with Yuan Yuan.

Various rigidity phenomena of CR maps between model CR manifolds have been investigated in literature. Due to the complexity of CR manifolds, such phenomena are in general not expected for general CR manifolds unless additional conditions are imposed. In this talk, we shall discuss a rigidity result for CR maps into hyperquadrics by imposing some geometric condition. More precisely, we show that all CR immersions from smooth Levi-nondegenerate hypersurfaces into hyperquadrics with vanishing CR second fundamental forms are necessarily linear fractional.