MWAA 2018
Indiana University, October 5–7th

Friday		
15:30–16:00	**Coffee**	
16:00–17:00	**K. McLaughlin**	Asymptotic analysis of Riemann-Hilbert problems, with applications to integrable nonlinear wave equations
Saturday		
09:00–09:40	**Coffee**	
09:40–10:20	**B. Langowski**	Discrete harmonic analysis in the non-commutative setting
10:30–11:10	**K. Taylor**	Interior, dimension, and measure of algebraic sums of fractal sets and curves from the view point of Fourier analysis and projection theory
11:20–12:00	**J. Lebl**	Complex analysis with a real parameter and the Levi-flat Plateau problem
12:00–14:00	**Lunch**	
14:00–14:40	**K. Liechty**	Error bounds in Fourier extension approximations
14:50–15:30	**M. Girotti**	Rigorous asymptotics of a KdV soliton gas
15:30–16:00	**Coffee**	
16:00–16:40	**J. Brauchart**	Hyperuniformity in the compact setting: measuring the fine structure of a sequence of point sets on the sphere
16:50–17:30	**T. Stepaniuk**	Comparison of probabilistic and deterministic point sets on the unit sphere
Sunday		
09:00–09:50	**Posters & Coffee**	
10:00–10:40	**A. Legg**	Quadrature domains and equilibrium on the sphere
10:50–11:30	**D. Dauvergne**	Asymptotic zero distribution of random orthogonal polynomials
11:40–12:20	**M. Manolaki**	Optimal polynomial approximants of reciprocals of analytic functions

Plenary Talks

Johann Brauchart

Technische Universität Graz

Hyperuniformity in the compact setting: measuring the fine structure of a sequence of point sets on the sphere

Hyperuniformity was introduced by Torquato and Stillinger as a concept to measure the occurrence of "intermediate" order between crystalline order and total disorder. Such configurations X occur in jammed packings, in colloids, as well as in quasi-crystals. The main feature of hyperuniformity is the fact that local density fluctuations ("number variance") are of smaller order than for an i.i.d. random ("Poissonian") point configuration.

In recent work with Peter Grabner (Graz University of Technology), Wöden Kusner (Vanderbilt University) and Jonas Ziefle (University of Tübingen), we introduced the notion of hyperuniformity for sequences of finite point sets on the sphere. We identified three regimes of hyperuniformity. Several deterministically given point sets such as designs, QMC-designs, and certain energy minimising point sets exhibit hyperuniform behaviour.

We also considered hyperuniformity on the sphere for samples of point processes on the sphere.

Duncan Dauvergne

University of Toronto

Asymptotic zero distribution of random orthogonal polynomials

Joint work with Thomas Bloom

We consider the global behaviour of the zeros of random polynomials of the form

$$H_n(z) = \sum_{i=0}^{n} \xi_i q_i(z),$$

where the coefficients ξ_i are i.i.d. complex random variables and the q_i are orthogonal polynomials. When $q_i(z) = z^i$ and the ξ_i are i.i.d. complex Gaussians, it is a classical result that the zeros of H_n cluster uniformly around the unit circle as n approaches infinity. In this talk, I will discuss how and when this phenomenon extends to general orthogonal polynomials and general non-degenerate coefficient distributions.

Manuela Girotti
Colorado State University
Rigorous asymptotics of a KdV soliton gas

Joint work with Ken McLaughlin (CSU) and Tamara Grava (SISSA, Bristol)

We analytically study the long time and large space asymptotics of a new broad class of solutions of the KdV equation introduced by Dyachenko, Zakharov, and Zakharov. These solutions are characterized by a Riemann-Hilbert problem which we show arises as the limit $N \nearrow +\infty$ of a gas of N-solitons. We establish an asymptotic description for large times that is valid over the entire spatial domain, in terms of Jacobi elliptic functions.

Bartosz Langowski
Indiana University
Discrete harmonic analysis in the non-commutative setting

Ionescu, Magyar, and Wainger [1] proved L^2-boundedness of discrete singular Radon transforms along polynomial mappings P of arbitrary order in discrete nilpotent groups of step 2. More precisely, they proved the following.

Theorem. *Assume that $\mathbb{G}$ is a discrete nilpotent Lie group of step 2, K is a Calderón-Zygmund kernel and $A : \mathbb{Z} \to \mathbb{G}$ is a polynomial sequence. Let*

$$(Hf)(g) = \sum_{n \in \mathbb{Z}} K(n) f(A^{-1}(n) \cdot g), \qquad g \in \mathbb{G},$$

then

$$\|Hf\|_{L^2(\mathbb{G})} \lesssim \|f\|_{L^2(\mathbb{G})}.$$

The approach they used seems to make it indispensable to assume that the underlying Lie group is of step 2. The aim of our long-term program is to relax this restriction and prove the L^2-estimate for groups of arbitrary orders.

In the talk we report our recent progress connected with this problem. We begin with considering as a toy model the analogous question in the commutative setting of $\mathbb{Z}^d$ with the underlying group of Euclidean translations. The tricky part is that in all the arguments we avoid as long as it is possible the use of the Fourier transform methods. Consequently, we expect that it shall be later possible to transfer the reasonings to the non-commutative setup, where the application of the Fourier transform is very limited.

References

[1] A. D. Ionescu, A. Magyar, S. Wainger. Averages along Polynomial Sequences in Discrete Nilpotent Lie Groups: Singular Radon Transforms. *Advances in Analysis:* The Legacy of Elias M. Stein (2014), 146.

Jiří Lebl
Oklahoma State University
Complex analysis with a real parameter and the Levi-flat Plateau problem

Joint work with Alan Noell and Sivaguru Ravisankar

There exist analogues of many several complex variables results to the space $\mathbb{C}^n \times \mathbb{R}$. I will talk about, in particular, the extension of CR functions and its application to a certain solution of the so-called Levi-flat Plateau problem. A Levi-flat hypersurface is a real hypersurface foliated by complex hypersurfaces. The Plateau problem asks for such a hypersurface given a boundary.

Alan Legg
Purdue University Fort Wayne
Quadrature domains and equilibrium on the sphere

With the unit sphere envisioned as a conductor holding a unit positive charge, imagine placing various positive point charges onto the sphere to constitute an external field. What shape will the unit charge obtain in the presence of this external field? We will use complex analysis to relate the answer to quadrature domains.

Karl Liechty
DePaul University
Error bounds in Fourier extension approximations

Joint work with Jeff Geronimo

A truncated Fourier series is a very effective way to approximate smooth periodic functions, but if a function defined on an interval is not periodic, its nearest truncated Fourier series is not a good approximation near the endpoints of the interval due to the Gibbs phenomenon. One method to deal with this issue, known as Fourier extension or Fourier continuation, is to extend the function smoothly to one which is periodic on a slightly larger interval, and approximate by truncated Fourier series with a slightly larger period. I will discuss asymptotic error bounds for these truncated Fourier series as the number of Fourier modes becomes large. In particular I will discuss the issue of obtaining uniform bounds from a discrete L^2 construction.

Myrto Manolaki
University of South Florida

Optimal polynomial approximants of reciprocals of analytic functions

Joint work with Catherine Bénéteau, Oleg Ivrii, and Daniel Seco

Given a Hilbert space H of analytic functions on the unit disc and a function $f \in H$, a polynomial p_n is called an *optimal polynomial approximant* of order n of $1/f$ if p_n minimizes $\|pf - 1\|$ over all polynomials p of degree at most n. This notion was introduced to investigate the phenomenon of cyclicity in certain function spaces, including the classical Hardy, Bergman, and Dirichlet spaces. This talk will highlight similarities and differences between Taylor expansions and optimal polynomial approximants, focusing on their limiting behaviour on the unit circle.

Ken McLaughlin
Colorado State University

Asymptotic analysis of Riemann-Hilbert problems, with applications to integrable nonlinear wave equations

Recently there has appeared results providing the global asymptotic behavior of solutions of several nonlinear PDEs, for very general initial conditions. These equations include the focusing and defocusing nonlinear Schröedinger equation, as well as the derivative nonlinear Schröedinger equation, amongst a few others. The development of d-bar methods to assist with the asymptotic analysis of Riemann-Hilbert problems (joint work with Peter Miller) plays a crucial role in the analysis. If all goes well, the talk will explain the topics in the title, and how the d-bar operator is used to streamline some of the analysis.

Tetiana Stepaniuk
Technische Universität Graz

Comparison of probabilistic and deterministic point sets on the unit sphere

We make a comparison between certain probabilistic and deterministic point sets and show that some deterministic constructions (spherical t-designs, minimizing point-sets) are better or as good as probabilistic ones. In particular the asymptotic equalities for the discrete Riesz s-energy of N-point sequence of well separated t-designs on the unit sphere $\mathbb{S}^d \subset \mathbb{R}^{d+1}$, $d \geqslant 2$ are found.

Krystal Taylor
Ohio State University

Interior, dimension, and measure of algebraic sums of fractal sets and curves from the view point of Fourier analysis and projection theory

Joint work with Karoly Simon

It is a time honored and classic problem to ask for the properties of the algebraic sum $A + B$ given sets A and B in the Euclidean plane. We focus on the case when Γ is a piecewise $\mathcal{C}^2$ curve (such as the unit circle). There is a natural guess what the size (Hausdorff dimension, Lebesgue measure) of $A + \Gamma$ should be. We verify this under some natural assumptions. We also address the more difficult question: under which condition does the set $A + \Gamma$ have non-empty interior? The results have some surprising consequences for distance sets:

$$\Delta_x(A) := \{|x - y| : y \in A\},$$

where x is a fixed point and A is a fractal subset of $\mathbb{R}^d$ of sufficient Hausdorff dimension. The relation between structure within a fractal set (as measured by sufficient Hausdorff dimension or by the existence of geometric configurations within) and the Fourier decay of a measure supported on said set is implicit.

Poster Presentations

Hanan Aljubran
IUPUI

Joint work with Maxim Yattselev

Let $\{\varphi_i\}_{i=0}^{\infty}$ be a sequence of orthonormal polynomials on the unit circle with respect to a positive Borel measure μ that is symmetric with respect to conjugation. Asymptotic behavior of the expected number of real zeros, say $\mathbb{E}_n(\mu)$, of random polynomials

$$P_n(z) := \sum_{i=0}^{n} \eta_i \varphi_i(z),$$

where $\eta_0, \ldots, \eta_n$ are i.i.d. standard Gaussian random variables is studied. When μ is the arclength measure such polynomials are called Kac polynomials and it was shown by Wilkins that $\mathbb{E}_n(|d\xi|)$ admits an asymptotic expansion of the form

$$\mathbb{E}_n(|d\xi|) \sim \frac{2}{\pi}\log(n+1) + \sum_{p=0}^{\infty} A_p(n+1)^{-p}$$

(Kac himself obtained the leading term of this expansion). We generalize the result of Wilkins to the case where μ is absolutely continuous with respect to arclength measure and its Radon-Nikodym derivative extends to a holomorphic non-vanishing function in some neighborhood of the unit circle. In this case $\mathbb{E}_n(\mu)$ admits an analogous expansion with coefficients the A_p depending on the measure μ for $p \geqslant 1$ (the leading order term and A_0 remain the same).

Ahmad Barhoumi
IUPUI

Joint work with Maxim Yattselev

Asymptotic behavior of polynomials $Q_n(z)$ satisfying non-Hermitian orthogonality relations

$$\int_\Delta s^k Q_n(s)\rho(s)ds = 0, \quad k \in \{0, \ldots, n-1\},$$

where $\Delta := [-a, a] \cup [-ib, ib]$, $a, b > 0$, and $\rho(s)$ is a Jacobi-type weight is investigated. The primary motivation for this work is study of the convergence properties of the Padé approximants to functions of the form

$$f(z) = (z-a)^{\alpha_1}(z-ib)^{\alpha_2}(z+a)^{\alpha_3}(z+ib)^{\alpha_4},$$

where the exponents $\alpha_i \notin \mathbb{Z}$ add up to an integer.

Andrei Prokhorov
IUPUI

Joint work with Alexander Its

The goal of this paper is to develop tools for asymptotic analysis of isomonodromic tau functions. In different previous works the tau functions for certain Painlevé equations was related to corresponding classical actions. We continue this ideology and relate the tau functions to classical action for all Painlevé equations and system of Schlesinger equations. Such correspondence is useful, since it provides differential identities required for asymptotic analysis. We also deduce similar differential identities for the general isomonodromic deformations from the work by Its, Lisovyy, Prokhorov. During our computations we managed to get the Hamiltonian structure for considered isomonodromic deformations. We conjecture the Hamiltonian structure for general isomonodromic deformations.

Alexey Sukhinin
North Dakota State University

The systems of the coupled focusing nonlinear Schrödinger equations on $(2+1)$D unbounded domain are considered. New classes of Vector Solitons were obtained as an extension of the well-known Townes Soliton. In the non-resonant model, it was observed that the Solitons appear as the threshold between decaying and collapsing solutions. This is not the case for the resonant model. A collapse of a single Gaussian beam due to self-focusing is a fundamental phenomenon in nonlinear optics and is well-studied. It has an important application in the area of filamentation. In particular, collapse leads to ionization of the air and creation of plasma channel. In this work, new types of vector singularity formations were obtained both at resonance and non-resonance.

Aaron Yeager
Oklahoma State University

Let $\{\varphi_k\}_{k=0}^{\infty}$ be a sequence of orthonormal polynomials on the unit circle with respect to a probability measure μ. We study the variance of the number of zeros of random linear combinations of the form

$$P_n(z) = \sum_{k=0}^{n} \eta_k \varphi_k(z),$$

where $\{\eta_k\}_{k=0}^{n}$ are complex-valued random variables. Assuming that $d\mu(\theta) = w(\theta)d\theta$, with $w(\theta) \geqslant c > 0$ for $\theta \in [0, 2\pi)$, and the distribution for each η_k satisfies certain uniform bounds for the fractional and logarithmic moments, we show that the variance of the number of zeros of P_n in annuli that contain the unit circle is at most of the order $n\sqrt{n \log n}$ as $n \to \infty$. When μ is symmetric with respect to conjugation and in the Nevai class, and $\{\eta_k\}_{k=0}^{n}$ are i.i.d. complex-valued standard Gaussian, we prove a formula for the limiting value of variance of the number of zeros of P_n in annuli that do not contain the unit circle.